INVESTIGATION OF TEMPERATURE FIELDS IN A TWO-LAYER SYSTEM WITH A PULSED HEAT – RADIATION EFFECT

L. S. Slobodkin, T. T. Shinkevich, and S. G. Krasnich

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Results are presented of an analytical and experimental investigation of the temperature field in a coating—substrate system under a pulsed heat radiation effect.

Complex physicochemical transformations occur in polymer-filled composites (enameled and compound coatings) during drying and solidifying. Evaporation of the solvents occurs in the first stage at relatively low temperatures. As the temperature rises further, formation of a three-dimensional polymer structure occurs in the coating—substrate system at the same time as the removal of the solvents. A perfectly specific orientation of the polymer molecules hence occurs in the whole thickness of the sealing film, which is essentially dependent on the thermophysical parameters of the process and governs the quality of the coating. It has been established earlier that both the kinetics of solvent removal and the specifics of the temperature field development in the system, especially during its nonstationary heating period [1], influences the process under consideration. It has been established experimentally that the solidification processes have been intensified successfully during a pulsed heat radiation effect on some polymer coatings, and the quality of the polymer composites obtained has been raised [2]. Therefore, from the thermophysical viewpoint it is meaningful to study the peculiarities of temperature field development in multilayer systems in order to examine the thermal solidification processes of coatings under a pulsed energy effect. It is hence necessary to note the recently published paper [3] about the method and the fundamental tables compiled to determine the nonstationary temperatures in flat bodies under a pulsed radiant effect.

The experimental determination of temperature fields in thin-layered systems, each of which can have several tens of microns thickness, is quite difficult and even impossible in a number of practical cases. At the same time, taking account of the actual periods of a pulsed energy effect and the relatively high radiant energy flux densities incident on the object, it is necessary to have data about the development of temperature fields within small time segments calculated in fractions of a second.

In this connection, as well as with the experimental investigation conducted, it turned out to be expedient to consider the formulation of the problem, known from [5], taking account of the function $q(\tau)$ (the radiant flux acting on the surface) described by a Fourier series. Let us have two infinite plates of thickness l_1 and l_2 with different thermophysical coefficients. The initial temperature of the plates is identical and equal to t_0. A time-varying heat flux $q(\tau)$ is supplied to one of the surface of the thin-layered system, while the other surface is heat insulated. An ideal thermal contact exists on the contact boundary of the plates. It is required to find the temperature distribution in the system. Let us note than an analogous problem for a three-layer coating—substrate—coating system reduces to the problem formulated in the case of a symmetric energy effect.

The heat flux $q(\tau)$ can be represented graphically as presented in Fig. 1. Analytically the flux can be described by a Fourier series [4]:

$$q(\tau) = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos \omega\tau, \tag{1}$$

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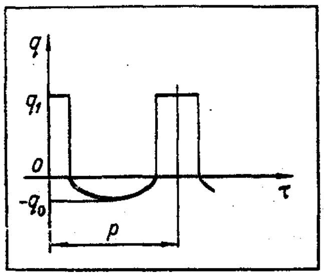

Fig. 1. Graphical picture of the pulsed radiant (q_1)-convective (q_0) heat flux in the system.

where

$$A_m = \frac{4}{p}\left\{q_1 \int_0^{p_1/2} \cos\omega\tau d\tau + q_0 \int_{p_1/2}^{p/2} \cos\left[\frac{\pi}{p_2}\left(\tau + \frac{p_2 - p_1}{2}\right)\right]\cos\omega\tau d\tau\right\},$$

$$m = 0,\ 1,\ 2,\ \ldots,$$

$$\omega = \frac{2m\pi}{p};\ p = p_1 + p_2;\ q_0,\ q_1 = \text{const}.$$

Then

$$\frac{\partial t_1(x,\ \tau)}{\partial\tau} = a_1\frac{\partial^2 t_1(x,\ \tau)}{\partial x^2} \quad (-l_1 < x < 0;\ \tau > 0),$$

$$\frac{\partial t_2(x,\ \tau)}{\partial\tau} = a_2\frac{\partial^2 t_2(x,\ \tau)}{\partial x^2} \quad (0 < x < l_2,\ \tau > 0), \tag{2}$$

$$t_1(x,\ 0) = t_2(x,\ 0) = t_0 = \text{const}, \tag{3}$$

$$\lambda_1\frac{\partial t_1(-l_1,\ \tau)}{\partial x} - q(\tau) = 0, \tag{4}$$

$$t_1(0,\ \tau) = t_2(0,\ \tau), \tag{5}$$

$$\lambda_1\frac{\partial t_1(0,\ \tau)}{\partial x} = \lambda_2\frac{\partial t_2(0,\ \tau)}{\partial x}, \tag{6}$$

$$\frac{\partial t_2(l_2,\ \tau)}{\partial x} = 0. \tag{7}$$

Two solutions of the problem have been obtained: 1) for the first stage of system heating (for small values of the time), and 2) for the second stage (for large values of the time).

Applying the Laplace transform to (2) and taking account of (3)-(7), we obtain the solution for the transforms in the following form:

$$t_1(x,\ s) - \frac{t_0}{s}$$

$$= \frac{A_0\left(K_\lambda \operatorname{ch}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{ch}\sqrt{\frac{s}{a_1}}\,x - K_a^{\frac{1}{2}} \operatorname{sh}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{sh}\sqrt{\frac{s}{a_1}}\,x\right)}{2\lambda_1 s\sqrt{\frac{s}{a_1}}\left[K_\lambda \operatorname{ch}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{sh}\sqrt{\frac{s}{a_1}}\,l_1 + K_a^{\frac{1}{2}} \operatorname{sh}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{ch}\sqrt{\frac{s}{a_1}}\,l_1\right]}$$

$$+ \sum_{m=1}^{\infty}\frac{A_m s\left(K_\lambda \operatorname{ch}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{ch}\sqrt{\frac{s}{a_1}}\,x - K_a^{\frac{1}{2}} \operatorname{sh}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{sh}\sqrt{\frac{s}{a_1}}\,x\right)}{\lambda_1\sqrt{\frac{s}{a_1}}(s^2+\omega^2)\left[K_\lambda \operatorname{ch}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{sh}\sqrt{\frac{s}{a_1}}\,l_1 + K_a^{\frac{1}{2}} \operatorname{sh}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{ch}\sqrt{\frac{s}{a_1}}\,l_1\right]}; \tag{8}$$

$$t_2(x,\ s) - \frac{t_0}{s}$$

$$= \frac{A_0 \operatorname{ch}\sqrt{\frac{s}{a_2}}\,(l_2 - x)}{2\lambda_2 s\sqrt{\frac{s}{a_1}}\left[K_\lambda \operatorname{ch}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{sh}\sqrt{\frac{s}{a_1}}\,l_1 + K_a^{\frac{1}{2}} \operatorname{sh}\sqrt{\frac{s}{a_2}}\,l_2 \operatorname{ch}\sqrt{\frac{s}{a_1}}\,l_1\right]}$$

$$+\sum_{m=1}^{\infty}\frac{A_m s\,\mathrm{ch}\sqrt{\frac{s}{a_2}}(l_2-x)}{\lambda_2\sqrt{\frac{s}{a_1}}(s^2+\omega^2)\left[K_\lambda\,\mathrm{ch}\sqrt{\frac{s}{a_2}}l_2\,\mathrm{sh}\sqrt{\frac{s}{a_1}}l_1+K_a^{\frac{1}{2}}\mathrm{sh}\sqrt{\frac{s}{a_2}}l_2\,\mathrm{ch}\sqrt{\frac{s}{a_1}}l_1\right]}.$$

Let us use the decomposition theorem. Equating denominators of the solution (8) (the expressions in the square brackets) to zero, we obtain a characteristic equation of the form

$$K_\lambda\,\mathrm{tg}\,\mu+K_a^{\frac{1}{2}}\,\mathrm{tg}\,K_a^{\frac{1}{2}}K_l^{-1}\mu=0. \tag{9}$$

After going from the transform to the original, we obtain the following solution:

$$t_1(x,\tau)-t_0=\varphi(\tau)+\sum_{n=1}^{\infty}\frac{A_0 l_1^2}{\lambda_1\mu_n^2\nu_n}C_n\exp\left(-\mu_n^2\frac{a_1\tau}{l_1^2}\right)$$

$$+\sum_{m=1}^{\infty}\left\{B_m+\frac{A_m}{2\lambda_1\psi_m^{\pm}}\left(K_\lambda\,\mathrm{ch}\sqrt{\frac{\pm i\omega}{a_2}}\,l_2\,\mathrm{ch}\sqrt{\frac{\pm i\omega}{a_1}}\,x\right.\right.$$

$$\left.-K_a^{\frac{1}{2}}\,\mathrm{sh}\sqrt{\frac{\pm i\omega}{a_2}}\,l_2\,\mathrm{sh}\sqrt{\frac{\pm i\omega}{a_1}}\,x\right)\exp(\pm i\omega\tau)$$

$$\left.+\sum_{n=1}^{\infty}\frac{2A_m\mu_n^2}{\lambda_1 l_1^2\left(\mu_n^4\frac{1}{l_1^4}+\frac{\omega^2}{a_1^2}\right)\nu_n}C_n\exp\left(-\mu_n^2\frac{a_1\tau}{l_1^2}\right)\right\}; \tag{10}$$

$$t_2(x,\tau)-t_0=\varphi(\tau)+\sum_{n=1}^{\infty}\frac{A_0 l_1^2}{\lambda_2\mu_n^2\nu_n}\cos K_a^{\frac{1}{2}}\mu_n\left(K_l^{-1}-\frac{x}{l_1}\right)$$

$$\times\exp\left(-\mu_n^2\frac{a_1\tau}{l_1^2}\right)+\sum_{m=1}^{\infty}\left\{B_m+\frac{A_m}{2\lambda_2\psi_m^{\pm}}\,\mathrm{ch}\sqrt{\frac{\pm i\omega}{a_2}}(l_2-x)\exp(\pm i\omega\tau)\right.$$

$$+\sum_{n=1}^{\infty}\frac{2A_m\mu_n^2}{\lambda_2 l_1^2\left(\mu_n^4\frac{1}{l_1^4}+\frac{\omega^2}{a_1^2}\right)\nu_n}\cos K_a^{\frac{1}{2}}\left(K_l^{-1}-\frac{x}{l_1}\right)$$

$$\left.\times\mu_n\exp\left(-\mu_n^2\frac{a_1\tau}{l_1^2}\right)\right\},$$

where

$$\varphi(\tau)=\frac{A_0}{2\left(\frac{\lambda_1 l_1}{a_1}+\frac{\lambda_2 l_2}{a_2}\right)}\left[\tau-\frac{l_1 l_2(K_\lambda l_2+l_1)}{2(K_\lambda a_2 l_1+a_1 l_2)}-\frac{K_\lambda a_2^2 l_1^3+a_1^2 l_2^3}{6a_1a_2(K_\lambda a_2 l_1+a_1 l_2)}\right];$$

$$\nu_n=K_a^{\frac{1}{2}}(K_\lambda l_2+l_1)\sin K_a^{\frac{1}{2}}K_l^{-1}\mu_n\sin\mu_n$$

$$-(K_\lambda l_1+K_a l_2)\cos K_a^{\frac{1}{2}}K_l^{-1}\mu_n\cos\mu_n;$$

$$\psi_m^{\pm}=\sqrt{\frac{\pm i\omega}{a_1}}\left(K_\lambda\,\mathrm{ch}\sqrt{\frac{\pm i\omega}{a_2}}\,l_1\,\mathrm{sh}\sqrt{\frac{\pm i\omega}{a_1}}l_1\right.$$

$$\left.+K_a^{\frac{1}{2}}\,\mathrm{sh}\sqrt{\frac{\pm i\omega}{a_2}}\,l_2\,\mathrm{ch}\sqrt{\frac{\pm i\omega}{a_1}}\,l_1\right);$$

$$C_n=K_\lambda\cos K_a^{\frac{1}{2}}K_l^{-1}\mu_n\cos\frac{x}{l_1}\mu_n+K_a^{\frac{1}{2}}\sin K_a^{\frac{1}{2}}K_l^{-1}\mu_n\sin\frac{x}{l_1}\mu_n;$$

$$B_m=\frac{2A_m a_1}{\lambda_2\omega^2\left[K_\lambda l_1\left(\frac{l_2^2}{a_2}+\frac{l_1^2}{3a_1}\right)+K_a l_2\left(\frac{l_2^2}{3a_2}+\frac{l_1^2}{a_1}\right)\right]};$$

and $\mu_n = i\sqrt{(s_n/a_1)}\,l_1$ are the roots of the characteristic equation (9).

The series containing $\exp(-(\mu_n^2)(a_1\tau/l_1^2))$ converge rapidly since the exponential function diminishes rapidly as μ_n increases and starting with some value $\tau'' > \tau'$ these series can be neglected. Hence, the solutions for the second stage of system heating will become

$$t_1(x,\tau)-t_0=\varphi(\tau)+\sum_{m=1}^{\infty}\left\{B_m+\frac{A_m}{2\lambda_1\psi_m^+}\left(K_\lambda \operatorname{ch}\sqrt{\frac{i\omega}{a_2}}\,l_2 \operatorname{ch}\sqrt{\frac{i\omega}{a_1}}\,x - K_a^{\frac{1}{2}}\operatorname{sh}\sqrt{\frac{i\omega}{a_2}}\,l_2\operatorname{sh}\sqrt{\frac{i\omega}{a_1}}\,x\right)\exp(i\omega\tau)+\frac{A_m}{2\lambda_1\psi_m^-}\left(K_\lambda\operatorname{ch}\sqrt{\frac{-i\omega}{a_2}}\,l_2 \times \operatorname{ch}\sqrt{\frac{-i\omega}{a_1}}\,x-K_a^{\frac{1}{2}}\operatorname{sh}\sqrt{\frac{-i\omega}{a_2}}\,l_2\operatorname{sh}\sqrt{\frac{-i\omega}{a_1}}\,x\right)\exp(-i\omega\tau)\right\}; \tag{11}$$

$$t_2(x,\tau)-t_0=\varphi(\tau)+\sum_{m=1}^{\infty}\left\{B_m+\frac{A_m}{2\lambda_2\psi_m^+}\operatorname{ch}\sqrt{\frac{i\omega}{a_2}}\,(l_2-x)\exp(i\omega\tau)+\frac{A_m}{2\lambda_2\psi_m^-}\operatorname{ch}\sqrt{\frac{-i\omega}{a_2}}\,(l_2-x)\exp(-i\omega\tau)\right\}.$$

For small values of the time (the first stage of system heating), the quantity $\sqrt{(s/a_2)}\,l_2$ is large, and as is known, for large values of the argument (greater than 6.0) it is possible to set $\operatorname{sh}\sqrt{(s/a_2)}\,l_2 = \operatorname{ch}\sqrt{(s/a_2)}\,l_2 = (1/2)e^{\sqrt{(s/a_2)}l_2}$ [5] approximately, to the accuracy of the third decimal. Then we obtain after simple manipulations

$$t_1(x,s)-\frac{t_0}{s}=\frac{A_0}{4K_a^{\frac{1}{2}}\lambda_1}\left\{(K_\lambda-K_a^{\frac{1}{2}})\frac{\exp\left[-\sqrt{\frac{s}{a_1}}\,(l_1-x)\right]}{s\sqrt{\frac{s}{a_1}}}+(K_\lambda+K_a^{\frac{1}{2}})\frac{\exp\left[-\sqrt{\frac{s}{a_1}}\,(l_1+x)\right]}{s\sqrt{\frac{s}{a_1}}}\right\}$$
$$+\sum_{m=1}^{\infty}\frac{A_m\sqrt{a_2}}{2\lambda_1}\left\{(K_\lambda-K_a^{\frac{1}{2}})\frac{\sqrt{s}\exp\left[-\sqrt{\frac{s}{a_1}}\,(l_1-x)\right]}{s^2+\omega^2}+(K_\lambda+K_a^{\frac{1}{2}})\frac{\sqrt{s}\exp\left[-\sqrt{\frac{s}{a_1}}\,(l_1+x)\right]}{s^2+\omega^2}\right\}; \tag{12}$$

$$t_2(x,s)-\frac{t_0}{s}=\frac{A_0\sqrt{a_2}}{2\lambda_2}\,\frac{\exp\left[-\sqrt{s}\left(\frac{x}{\sqrt{a_2}}+\frac{l_1}{\sqrt{a_1}}\right)\right]}{s\sqrt{s}}+\sum_{m=1}^{\infty}\frac{A_m\sqrt{a_2}}{\lambda_2}\,\frac{\sqrt{s}\exp\left[-\sqrt{s}\left(\frac{x}{\sqrt{a_2}}+\frac{l_1}{\sqrt{a_1}}\right)\right]}{s^2+\omega^2}.$$

The first members of both expressions in (12) are tabulated transforms, and we use the theorem of multiplying transforms to go from the transform to the original under the sums. Finally, the approximate solution for the first stage of system heating is written as follows:

$$t_1(x,\tau)-t_0=\frac{A_0\sqrt{a_2\tau}}{2\lambda_1}\left[(K_\lambda-K_a^{\frac{1}{2}})\,i\operatorname{erfc}\frac{(l_1-x)}{2\sqrt{a_1\tau}}+(K_\lambda+K_a^{\frac{1}{2}})\times i\operatorname{erfc}\frac{(l_1+x)}{2\sqrt{a_1\tau}}\right]+\sum_{m=1}^{\infty}\frac{A_m\sqrt{a_2}}{2\lambda_1\sqrt{2\omega}}\left\{(K_\lambda-K_a^{\frac{1}{2}})\exp\left[-(l_1-x)\sqrt{\frac{\omega}{2a_1}}\right]\cos\left[\omega\tau-(l_1-x)\sqrt{\frac{\omega}{2a_1}}\right]\sin\left[\omega\tau\right.\right.$$

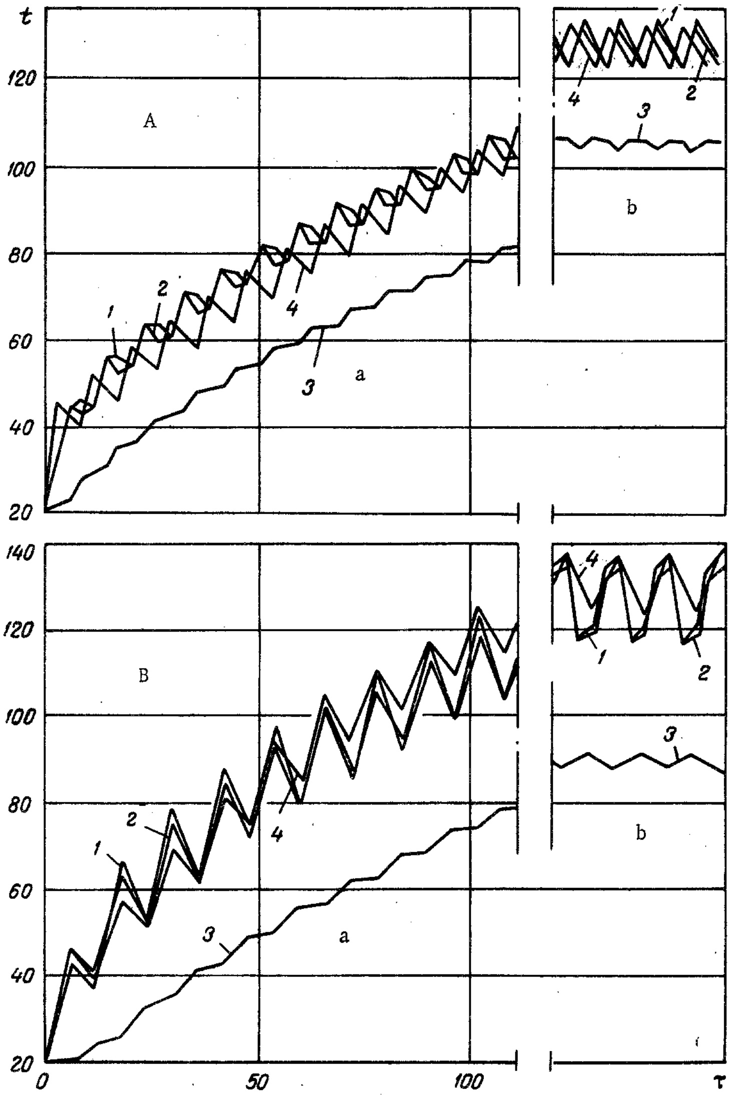

Fig. 2. Temperature field of a coating–substrate system for $p_1 : p_2 = 3 : 6$ sec (A) and $p_1 : p_2 = 6 : 6$ sec (B) during the heating period (a) and under quasistationary heating conditions (b): 1) surface; 2) layer junction; 3) substrate center; 4) layer junction according to experimental results t, °C; τ sec.

$$-(l_1-x)\sqrt{\frac{\omega}{2a_1}}\Bigg] + \left(K_\lambda + K_a^{\frac{1}{2}}\right)\exp\Bigg[-(l_1+x)$$

$$\times\sqrt{\frac{\omega}{2a_1}}\Bigg]\cos\Bigg[\omega\tau-(l_1+x)\sqrt{\frac{\omega}{2a_1}}\Bigg]\sin\Bigg[\omega\tau-(l_1+x)\sqrt{\frac{\omega}{2a_1}}\Bigg]\Bigg\}; \qquad (13)$$

$$t_2(x,\ \tau)-t_0=\frac{A_0\sqrt{a_2\tau}}{\lambda_2}\, i\,\mathrm{erfc}\,\frac{\left(\frac{x}{\sqrt{a_2}}+\frac{l_1}{\sqrt{a_1}}\right)}{2\sqrt{\tau}}$$

$$+\sum_{m=1}^{\infty}\frac{A_m\sqrt{a_2}}{\lambda_2\sqrt{2\omega}}\exp\left[-\left(\frac{x}{\sqrt{a_2}}+\frac{l_1}{\sqrt{a_1}}\right)\sqrt{\frac{\omega}{2}}\right]\cos\Bigg[\omega\tau$$

$$-\left(\frac{x}{\sqrt{a_2}}+\frac{l_1}{\sqrt{a_1}}\right)\sqrt{\frac{\omega}{2}}\Bigg]\sin\left[\omega\tau-\left(\frac{x}{\sqrt{a_2}}+\frac{l_1}{\sqrt{a_1}}\right)\sqrt{\frac{\omega}{2}}\right].$$

For quasistationary system heating conditions (11) permit determination of the temperature fluctuations over the system cross-section with respect to the mean temperature reached at the corresponding depths within the nonstationary period. To determine the time τ'' when it is necessary to use (11) in the computations, the members containing the exponential function $\exp(-\mu_n^2(a_1\tau/l_1^2))$ in (10) should be estimated. It is most convenient to do this by using an electronic computer.

Using the expressions (11), (13) obtained, curves of the heating kinetics and dynamics were constructed and analyzed for a coating—substrate system under pulsed thermal radiation heating and convective cooling. The coating in an actual system was glyptal enamel GF-916 with $l_1 = 30\ \mu$, while the substrate was a plate a technical ceramic with $2l_2 = 6000\,\mu$. The enamel was deposited on both substrate surfaces, however the conditions of the problem were satisfied by virtue of organizing symmetric heating and cooling of the system in the experiments. It should be noted that the enamel under consideration possesses high absorption ($K_\lambda \approx 310\ \mathrm{mm}^{-1}$) in the region of the maximum emitted IR energy of the quartz emitters used in the experiments and system heating occurred according to the coating—substrate scheme. The following mean values of the system thermophysical characteristics were taken in the computations [6]: $\bar{\lambda}_1 = 0.09$ W/m·deg; $\bar{\lambda}_2 = 0.5$ W/m·deg; $\bar{a}_1 = 0.52\cdot10^{-7}\ \mathrm{m^2/sec}$, $\bar{a}_2 = 7\cdot10^{-7}\ \mathrm{m^2/sec}$.

The relationships between the exposure p_1 and cooling p_2 periods were varied during the investigation. Hence, in order to comply with the conditions of the experiment about not exceeding the ultimate heating temperature of the heat-sensing system t_{max}, the change in the ratios $p_1 : p_2 = 1:1,\ 1:2,\ 1:3$ etc. was due to the change in the densities of the incident radiant fluxes q_1 within the limits $6\cdot10^3$–$1.5\cdot10^4$ W/m^2 and the cooling flux densities q_0—$1\cdot10^3$–$5\cdot10^3$ W/m^2.

Presented in Fig. 2A and B as an illustration are curves characterizing the temperature field development in ideal and actual two-layer systems. Changes in the temperature on the interface between the coating and the substrate, obtained experimentally for appropriate modal parameters of the process are shown. As follows from the figure, satisfactory agreement is observed between the analytical and experimental heating curves. The effect of the heating tempo and the change in the temperature values over the coating cross-section, evaluated as several degrees, on the kinetics and dynamics of the solidification processes of the kind of enamel under consideration has been established on the basis of analyzing the results of the analytical and experimental investigations.

Moreover, it turns out that the temperature drops relative to the mean values across the section of each layer with different thermophysical characteristics remain high in thin-layered systems in the pulsed mode not only under nonstationary heating conditions but also in the quasistationary period. As the duration of the radiation exposure pulse increases, the amplitudes of the fluctuations and the temperature drops over the system cross-section grow in an appropriate manner.

NOTATION

τ is the time;
t_1, t_2 are the temperature;
a_1, a_2 are the temperature conduction coefficients;
λ_1, λ_2 are the heat conduction coefficients;
l_1, l_2 are the layer thicknesses;
$K_a = a_1/a_2$;
$K_\lambda = \lambda_1/\lambda_2$;
$K_l^{-1} = l_2/l_1$.

Subscripts

1, 2 are the coating and the substrate;
q_1, q_0 are the radiant and convective heat flux densities;
p_1, p_2 are the periods of radiation heating and air blowing.

LITERATURE CITED

1. L. S. Slobodkin, I. S. Skorynina, and M. N. Barskaya, ÉP Électrothermy, No. 109, Scientific-Technical Collection. VNIIÉM (Informélektro) Division, Moscow, 31 (1971).
2. S. G. Krasnich, Heat and Mass Exchange in Drying and Heat Treating Processes [in Russian], Nauka i Tekhnika, Minsk (1970), p. 128.

3. P. A. Grigor'ev, V. A. Nuzhnyi, and B. V. Shibanov, Tables to Compute the Nonstationary Temperatures of Flat Bodies during Heating by Radiation [in Russian], Nauka, Moscow (1971).
4. M. G. Serebrennikov, Harmonic Analysis [in Russian], Gostekhizdat, Ogiz (1948).
5. A. V. Lykov, Theory of Heat Conductivity [in Russian], Vysshaya Shkola, Moscow (1967).
6. L. N. Novichenok, Heat and Mass Exchange in Non-Newtonian Fluids [in Russian], Nauka i Tekhnika, Minsk (1968), p. 239.